Practical theory for calculating the microwave instability threshold in electron storage rings

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Outline

- My perspective and motivation
- Theoretical approach
- Determining the instability threshold from the stable solutions
- Solution technique: example using the impedance of steady-state coherent synchrotron radiation
- Comparing theory and simulations for a broad-band resonator impedance
- Application to predicting longitudinal collective effects at the Advanced Photon Source (APS)
  - Longitudinal impedance model for the APS
  - Theoretical predictions for the microwave instability threshold
  - Comparisons of theory, simulations, and measurements for collective dynamics near the microwave instability threshold
  - Comparing the bunch lengthening and energy spread increase for currents at and above the microwave instability threshold
- Conclusions and possible extensions

My perspective & motivation

- The longitudinal microwave instability in storage rings has a long history
  - High-frequency perturbation that increases energy spread above threshold current
  - Theory for a coasting beam developed in the late 1960's [1,2]: Keil-Schnell criterion
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Basic Goal/Hope:
Combine Pelligrini-Wang-Krinsky analysis with mode-coupling intuition to obtain theory that

1. Is relatively easy to solve
2. Is more accurate than the Boussard theory
3. Provides some additional physical insight to MWI

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Theoretical approach

- Using phase space coordinates $(z, p_z) = (s - ct, -\delta)$, the Vlasov equation is

$$\left[\frac{\partial}{\partial s} + \frac{\partial \mathcal{H}}{\partial p_z}\frac{\partial}{\partial z} - \frac{\partial \mathcal{H}}{\partial z}\frac{\partial}{\partial p_z}\right] F(z, p_z; s) = 0$$

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- Change to action-angle variables of the static problem, $(z, p_z) \rightarrow (\Phi, I)$, and isolate the time-dependent perturbation due to the wakefields/impedance:

$$F(z, p_z; s) \rightarrow F(\Phi, \mathcal{I}; s) = F_0(\mathcal{I}) + F_1(\Phi, \mathcal{I}; s)$$

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Unperturbed motion:
rf focusing,
potential well distortion, ...

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Ratio of average to
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Longitudinal impedance
due to perturbation

Strength of collective effects

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- To make further analytic progress, we assume unperturbed motion can be approximated by simple harmonic motion (similar to [13] and [8]); then

$$\frac{\partial \mathcal{H}_0}{\partial \mathcal{I}} = \frac{\omega_s}{c} = \frac{\alpha_c \sigma_\delta}{\sigma_z}, \quad z = \sigma_z \sqrt{\frac{2\mathcal{I}}{\langle \mathcal{I} \rangle}} \cos\Phi, \quad F_0(\mathcal{I}) = \frac{e^{-\mathcal{I}/\langle \mathcal{I} \rangle}}{2\pi \langle \mathcal{I} \rangle}, \quad \text{and} \quad \langle \mathcal{I} \rangle = \sigma_z \sigma_\delta$$

[13] S. Petracca, Th. Demma, and K. Hirata. “Gaussian approximation of the bunch lengthening in electron storage rings with a typical wake function.” Phys. Rev. ST-Accel. Beams **8**, 074401 (2005).
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Closed form "solution"

- Isolate time dependence for the linear P.D.E.

$$F_1(\Phi, \mathcal{I}; s) = \tilde{F}_1(\Phi, \mathcal{I})e^{-i\Omega s/c}$$

Exponential growth if Im(Ω) > 0

Closed form "solution"

- Isolate time dependence for the linear P.D.E. and define bunching via

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Longitudinal bunching at frequency ck

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Longitudinal bunching at frequency ck

- Then, the Vlasov equation becomes

$$\left[-\frac{i\Omega}{c} + \frac{\omega_s}{c}\frac{\partial}{\partial \Phi}\right]\tilde{F}_1(\Phi, \mathcal{I}) = \frac{2I}{\gamma I_A}\frac{e^{-\mathcal{I}/\langle \mathcal{I}\rangle}}{2\pi \langle \mathcal{I}\rangle^2}\int dk\ \frac{Z_\parallel(k)}{kZ_0}\mathcal{B}(k)\sum_{n\neq 1} i^n n J_n\left(k\sigma_z\sqrt{2I/\langle I\rangle}\right)e^{in\Phi}$$

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$$\left[-\frac{i\Omega}{c} + \frac{\omega_s}{c}\frac{\partial}{\partial\Phi}\right]\tilde{F}_1(\Phi,\mathcal{I}) = \frac{2I}{\gamma I_A}\frac{e^{-\mathcal{I}/\langle\mathcal{I}\rangle}}{2\pi\langle\mathcal{I}\rangle^2}\int dk\,\frac{Z_\parallel(k)}{kZ_0}\mathcal{B}(k)\sum_{n\neq 1} i^n n J_n\left(k\sigma_z\sqrt{2I/\langle I\rangle}\right)e^{in\Phi}$$

- Solve for $\widetilde{F}_1$ by integrating over angle, then multiply by e^{-ikz} and integrate over phase space to get bunching equation

$$\boxed{\mathcal{B}(k) = \frac{2I}{\gamma I_A \alpha_c \sigma_\delta^2}\int dk'\,\frac{Z_\parallel(k')}{ik'Z_0}\mathscr{M}(\sigma_z k, \sigma_z k';\Omega)\mathcal{B}(k')}$$

With symmetric kernel: $\mathscr{M}(x,y;\Omega) = e^{-(x^2+y^2)/2}\left[e^{xy} - I_0(xy) + \sum_{n=1}^{\infty}\frac{2(\Omega/\omega_s)^2 I_n(xy)}{n^2-(\Omega/\omega_s)^2}\right]$

Closed form "solution"

- Isolate time dependence for the linear P.D.E. and define bunching via

$$F_1(\Phi, \mathcal{I}; s) = \tilde{F}_1(\Phi, \mathcal{I}) e^{-i\Omega s/c}$$

Exponential growth if Im(Ω) > 0

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- Similar equation but without synchrotron motion derived in [10-11] and used to justify the Boussard criterion by evaluating Ω → 0 limit

[10] J. M. Wang and C. Pelligrini. "On the condition for a single bunch high frequency fast blow-up," BNL-28O34 (1980).
[11] S. Krinsky and J. M. Wang. "Longitudinal instabilities of bunched beams subject to a non-harmonic rf potential," Particle Accel. **17**, 108 (1985).

Mode coupling for the microwave instability

- Within Sacherer's formalism [5], the microwave instability can be understood in terms of classical mode coupling:
  - At zero current, perturbations oscillate at harmonics of the synchrotron frequency, so that $\Omega = n\omega_s$ for integer n.
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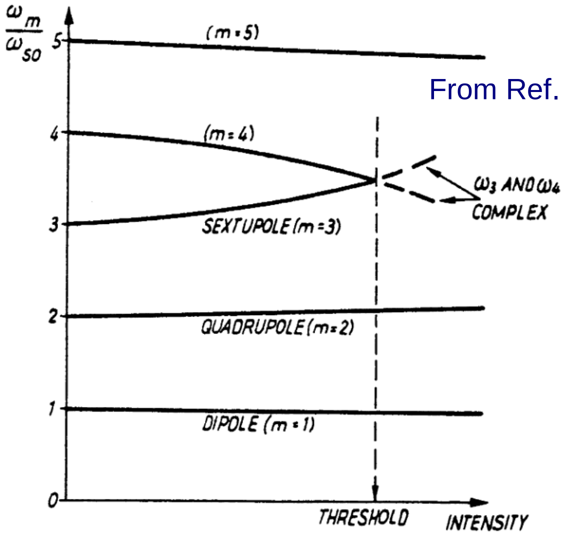

From Ref. [5]

Fig. 1 Coherent frequencies ω_m versus intensity

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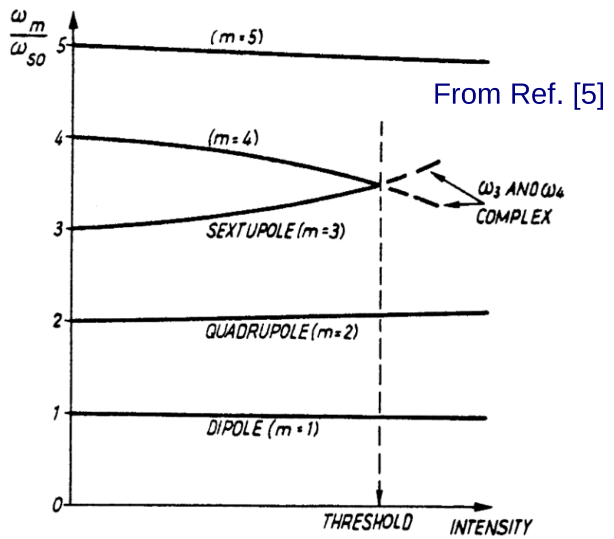

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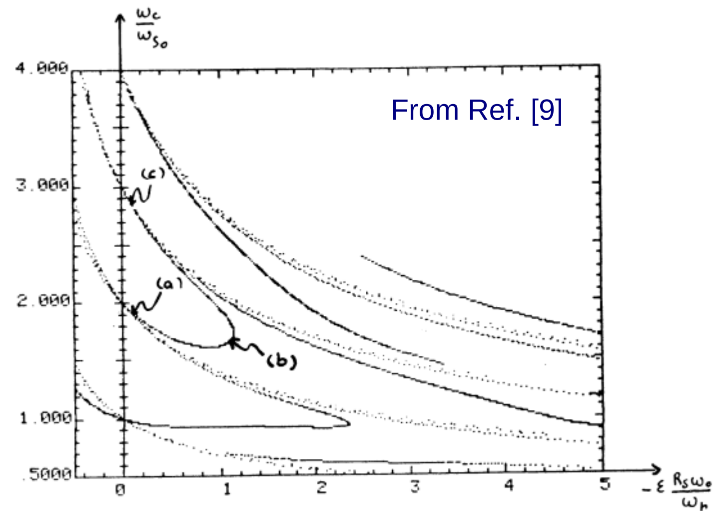

Fig. 20 Coherent-mode frequencies (m = 1 to 4) versus incoherent frequency shift (upper) and intensity parameter (lower)
a) Spectrum of the lowest radial quadrupole mode g_{22}
b) Coupling between quadrupole and sextupole modes at threshold
c) Spectrum of the lowest radial sextupole mode g_{33}

[5] F. Sacherer. “Bunch lengthening and microwave instability,” IEEE Trans. Nucl. Sci. **24**, 393 (1977)
[9] J. L. Laclare. “High current single bunch transverse instabilities at the ESRF: a new approach,” CERN accelerator school, pp 264 (1985).

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- One can use this to identify the lowest current at which stable oscillations cease to exist, which in turn gives the threshold current for mode coupling and the microwave instability.

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Example: steady-state CSR impedance

- CSR impedance for bending radius ρ is $Z_{\|}(k) = Z_0 \frac{\Gamma(2/3)}{3^{1/3}} \frac{\sqrt{3} + \mathrm{sgn}(k) i}{2} |k\rho|^{1/3}$
- Vlasov stability can be expressed in terms of a single dimensionless parameter, $\xi_{\mathrm{CSR}} \equiv \frac{I(\rho/\sigma_z)^{1/3}}{\gamma I_A \alpha_c \sigma_\delta^2}$, and the microwave instability threshold [12] has $\xi_{\mathrm{CSR}}^{\mathrm{thresh}} \approx 0.5$

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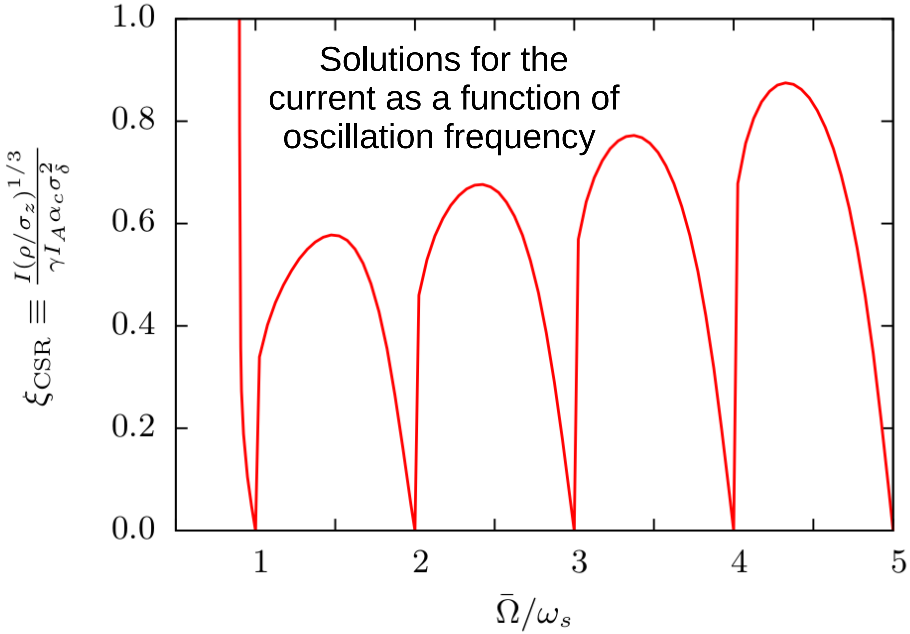

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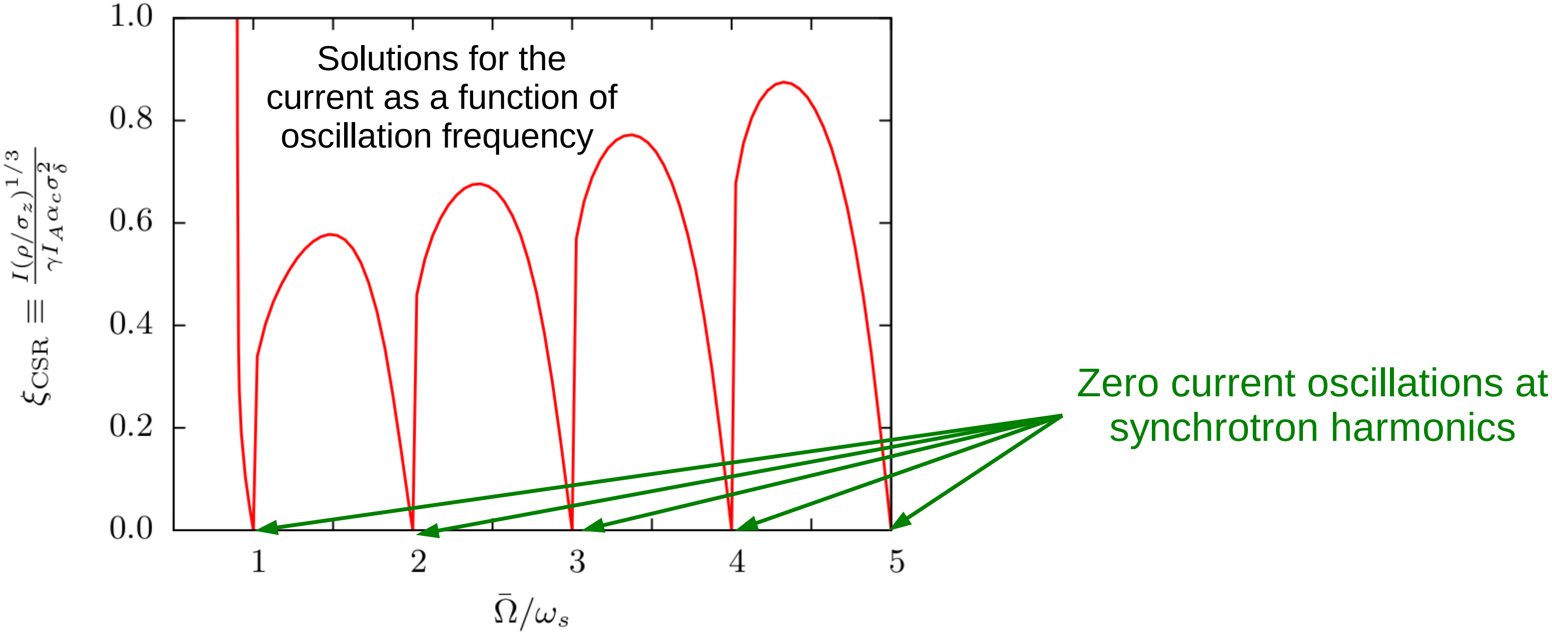

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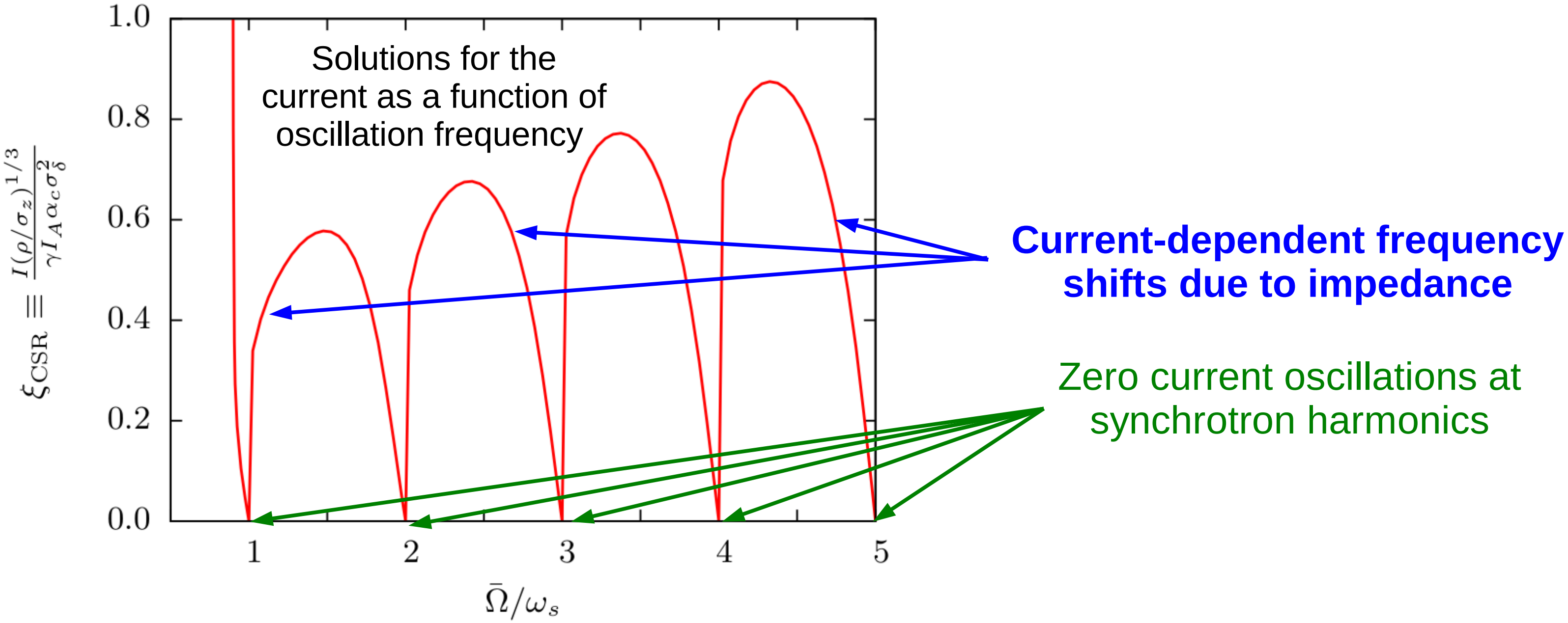

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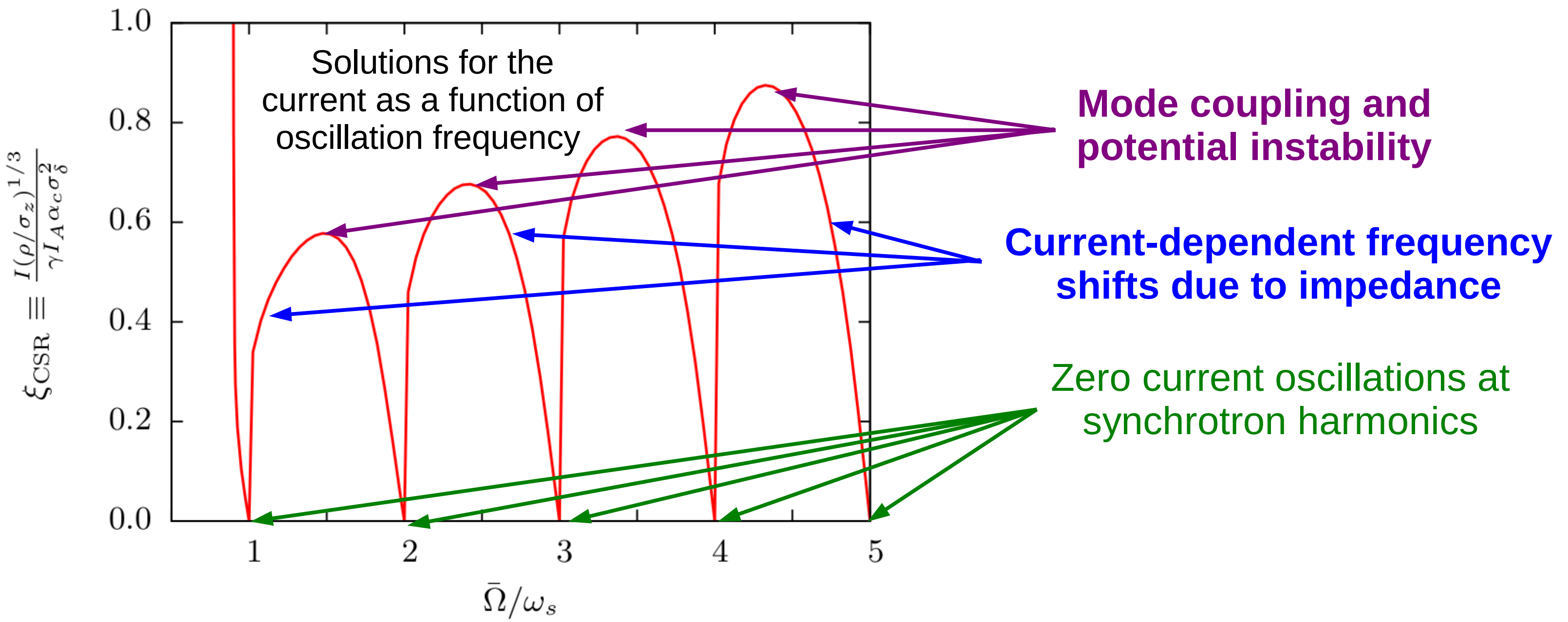

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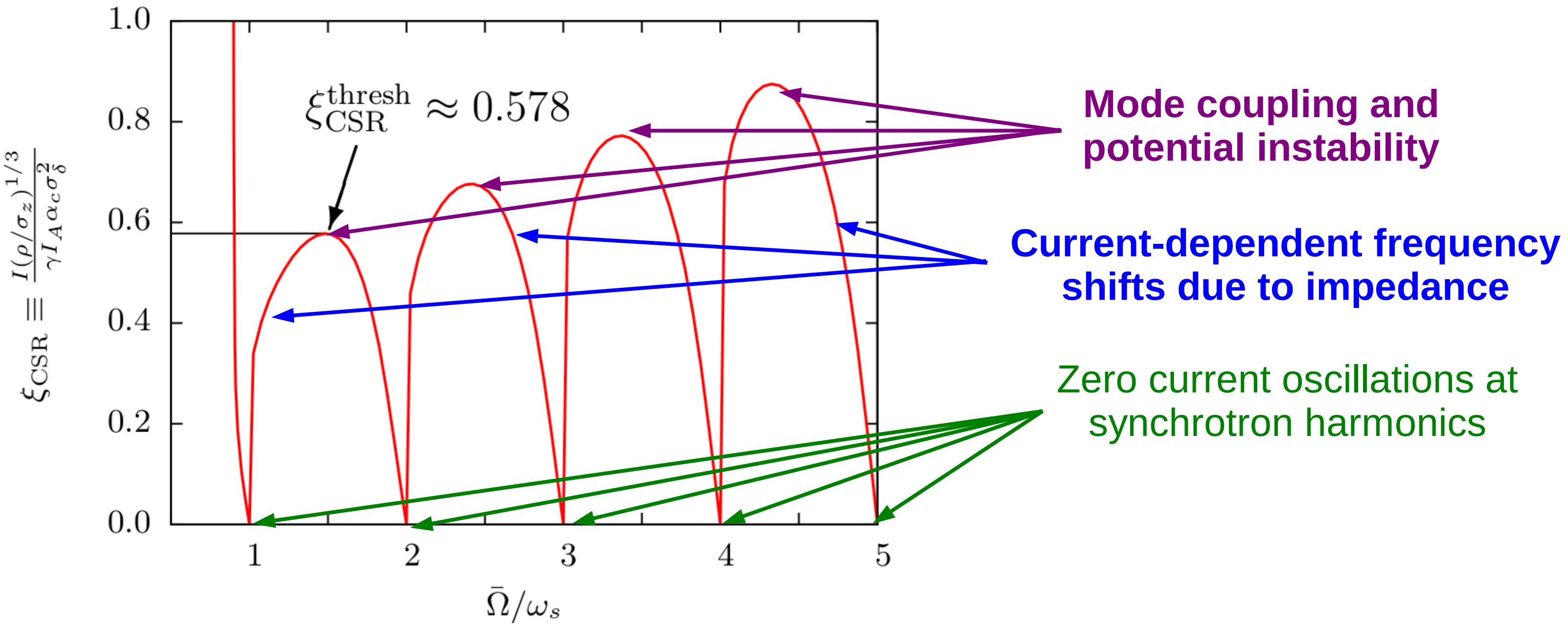

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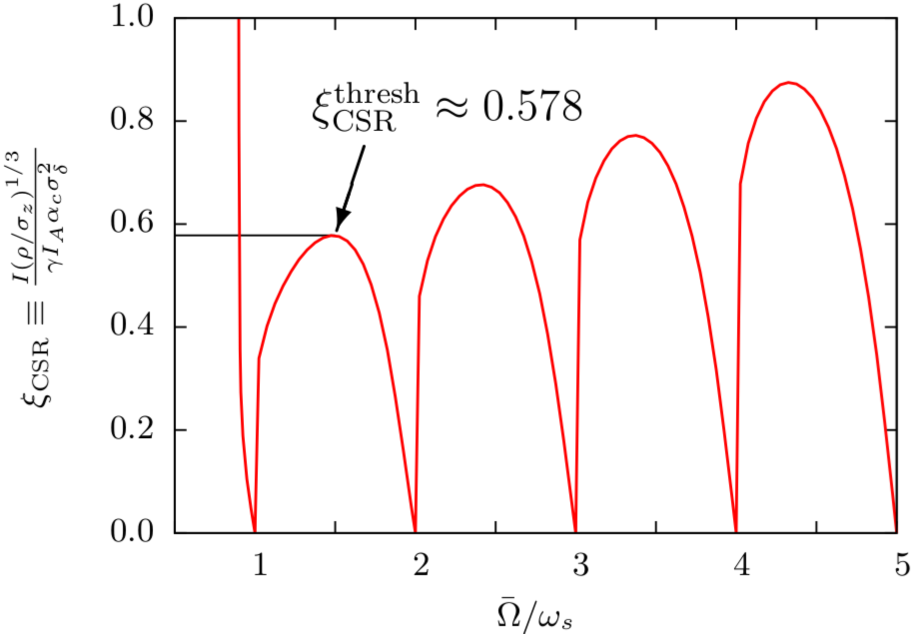

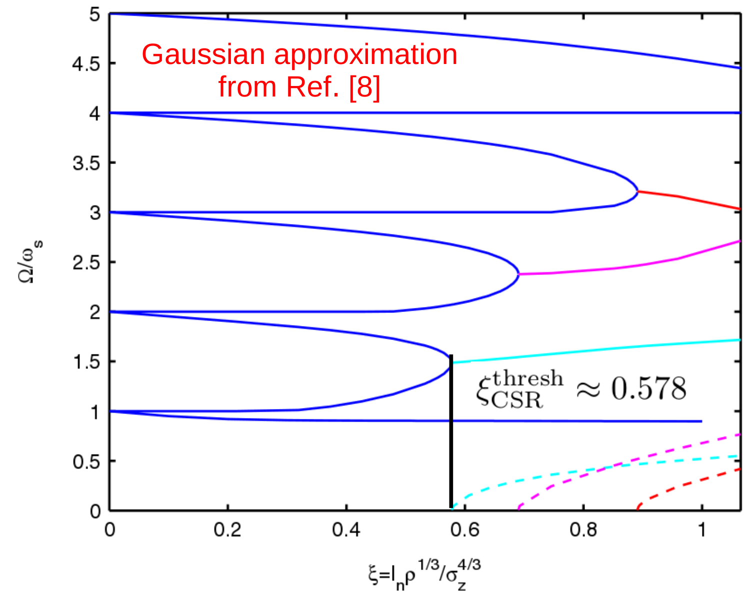

[12] K.L.F. Bane, Y. Cai, and G. Stupakov. "Threshold studies of the microwave instability in electron storage rings" PRST-AB **13**, 104402 (2010).
[8] Y. Cai. "Linear theory of microwave instability in electron storage rings." Phys. Rev ST Accel. Beams **14**, 061002 (2011)

Predictions for a broad-band resonator

- For the broad-band resonator impedance $Z_{\|}(k) = \frac{\omega_r R_s ck}{\omega_r ck + iQ\left[\omega_r^2 - (ck)^2\right]}$, Vlasov stability is determined by three dimensionless parameters:

$$\text{Frequency}: \ \nu_r = \frac{\omega_r \sigma_z}{c}, \qquad \text{Strength}: \ \xi_{\text{BBR}} = \frac{4\pi\nu_r I R_s}{\gamma I_A \alpha_c \sigma_\delta^2 Z_0}, \qquad \text{Quality factor}: \ Q \to 1$$

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$$\int dp_z \ F_0(z,p_z) = \rho_0(z) = \rho(0)e^{-V_0(z)/\alpha_c\sigma_\delta^2}, \quad V_0(z) = \frac{\omega_s^2}{2\alpha_c}z^2 - \frac{2I}{I_A}\int d\hat{z}dk \ \rho_0(\hat{z})e^{ik(z-\hat{z})}\frac{Z_{\|}(k)}{ikZ_0}$$

[14] J. Haïssinski, "Exact longitudinal equilibrium distribution of stored electrons in the presence of self-fields," Il Nuovo Cimento B **18**, 72 (1973).

Predictions for a broad-band resonator

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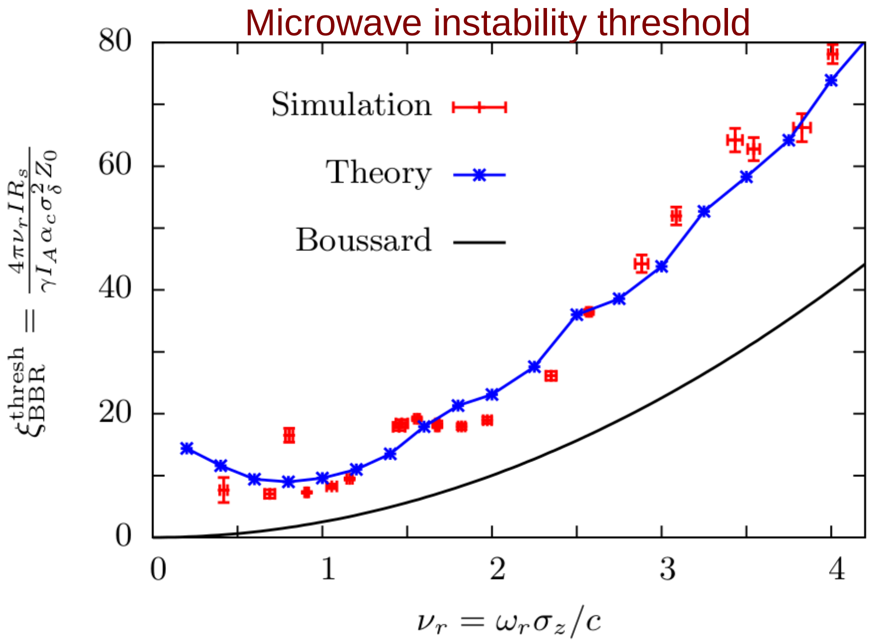

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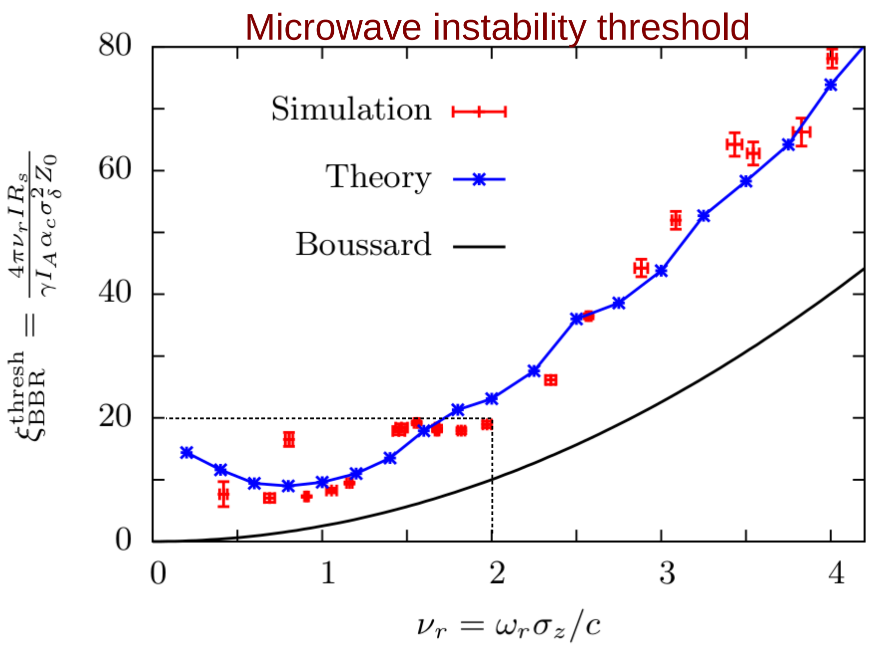
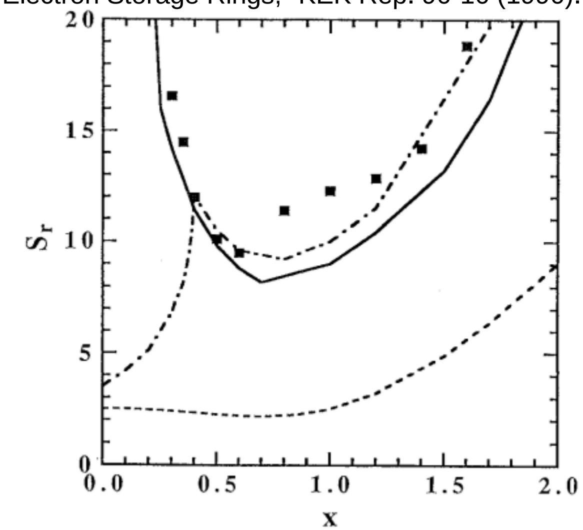


[7] K. Oide and K. Yokoya. “Longitudinal Single-Bunch Instability in Electron Storage Rings,” KEK Rep. 90-10 (1990).



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Predictions and measurements of longitudinal collective effects at the Advanced Photon Source (APS)

Impedance modeling at the APS

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  - Five different bellows configurations (240 total in ring)
  - Flange gaps (480 total; 2 mm wide, 3 mm deep)
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  - Resolves frequencies up to $f_{\text{max}} \sim c/(2\pi\sigma_b) \sim 50$ GHz
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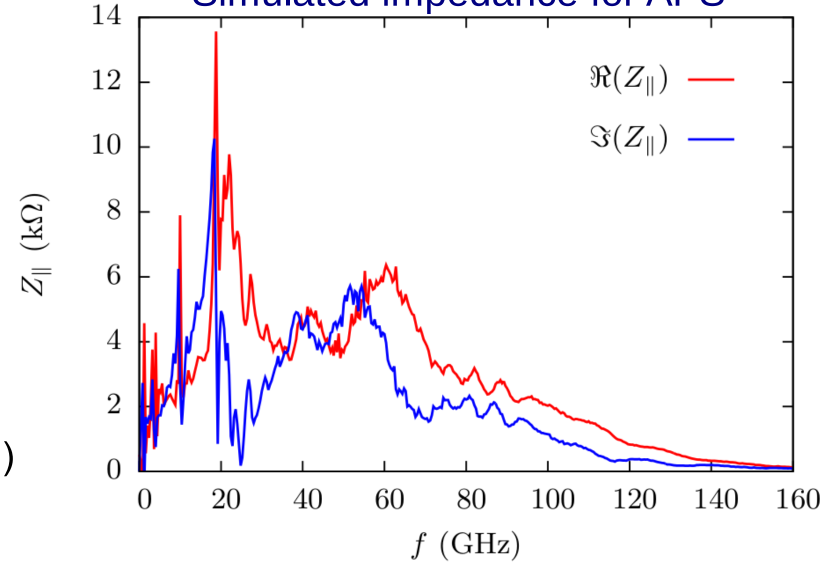

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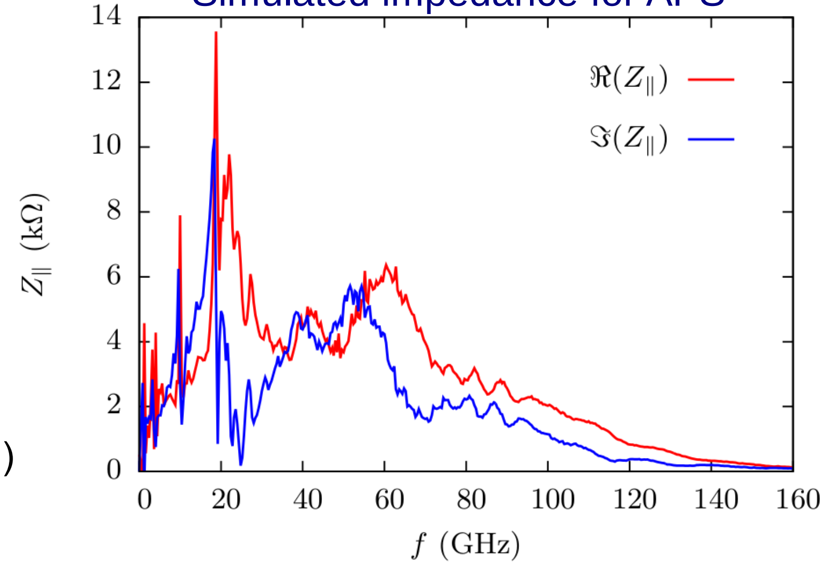
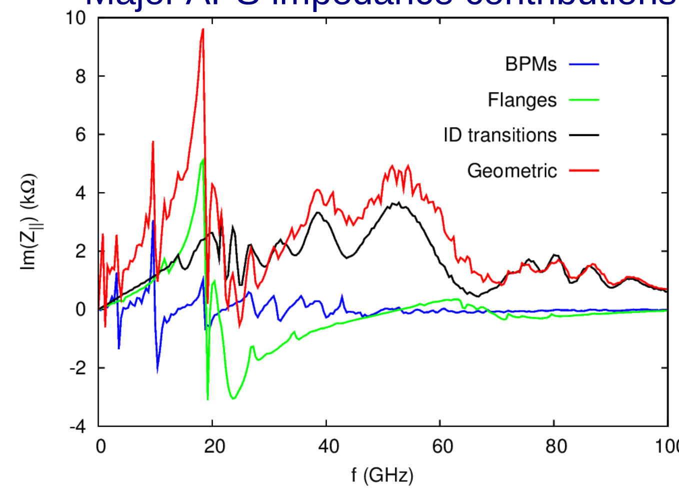


Major APS impedance contributions



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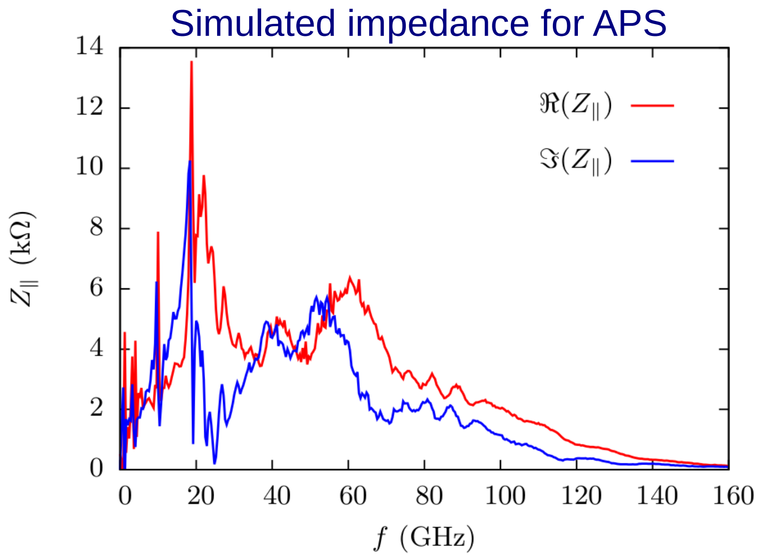
Simulating collective effects at the APS



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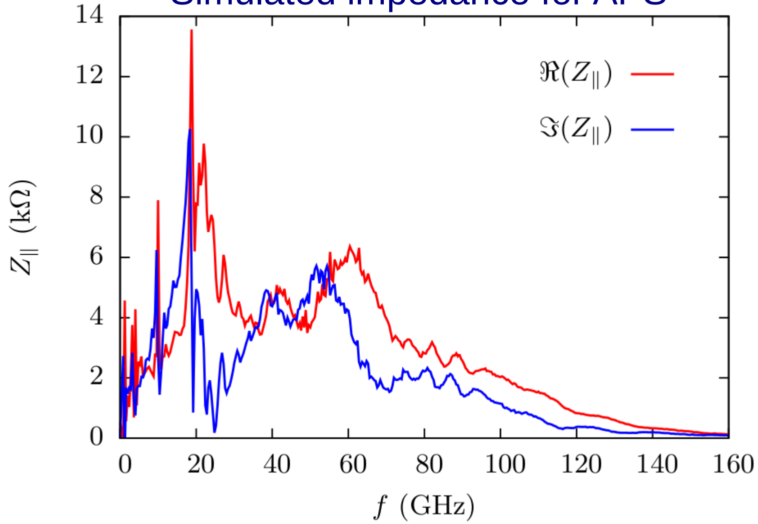

Other APS parameters for theory & tracking

Parameter	Symbol	Value
Beam energy	γmc^2	7 GeV
Momentum compaction	α_c	2.82×10^{-4}
Zero-current energy spread	$\sigma_{\delta,0}$	0.096 %
Rf voltage	V_{rf}	9 MV
Zero-current bunch length	$\sigma_{z,0}$	6.10 mm
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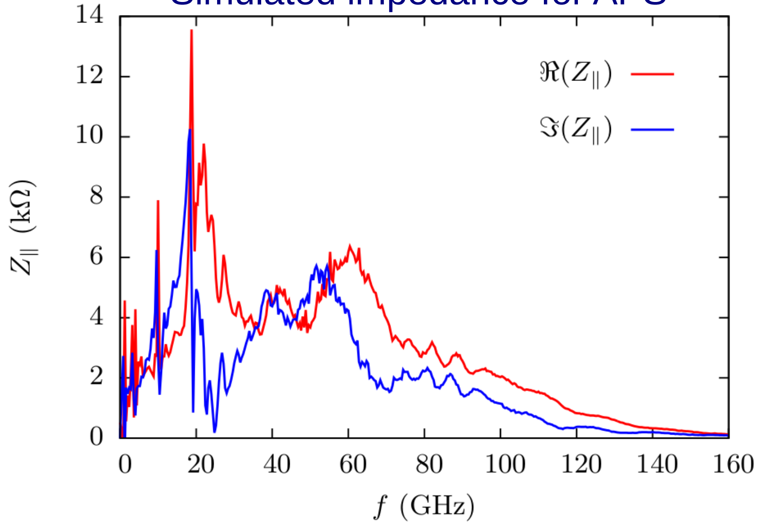

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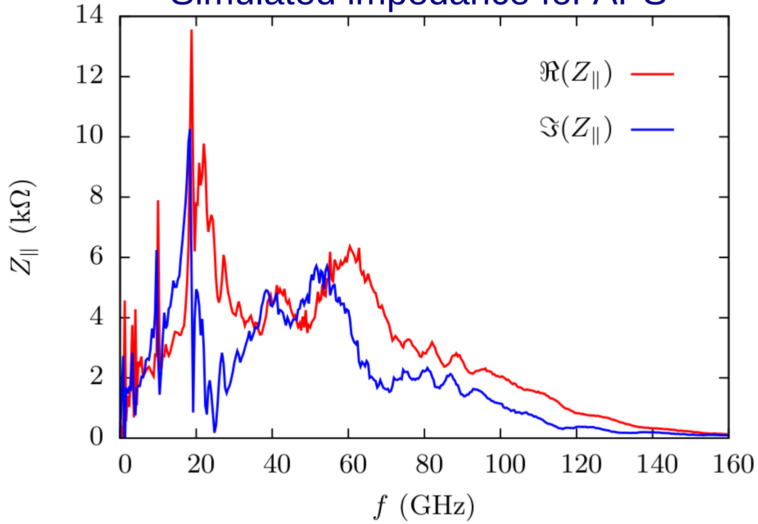

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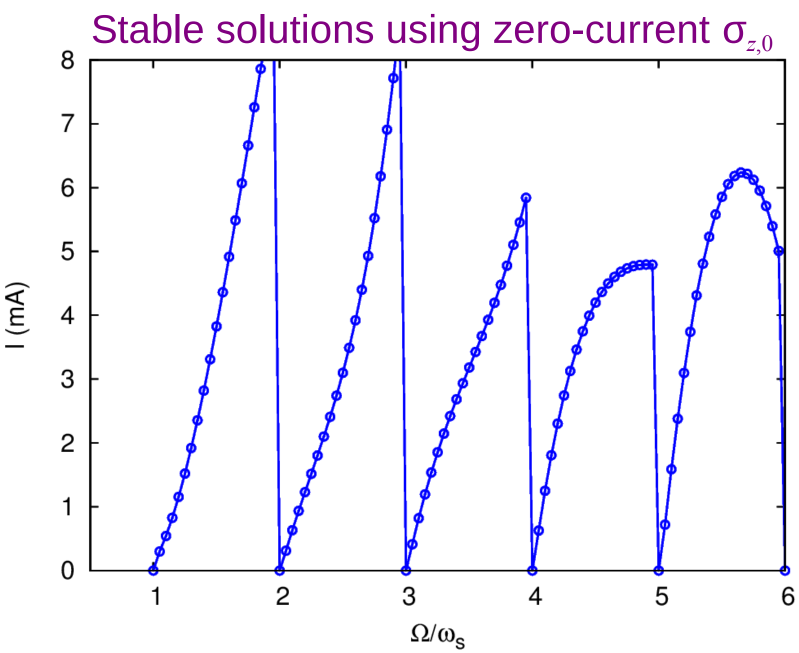
Applying the mode coupling theory to the APS



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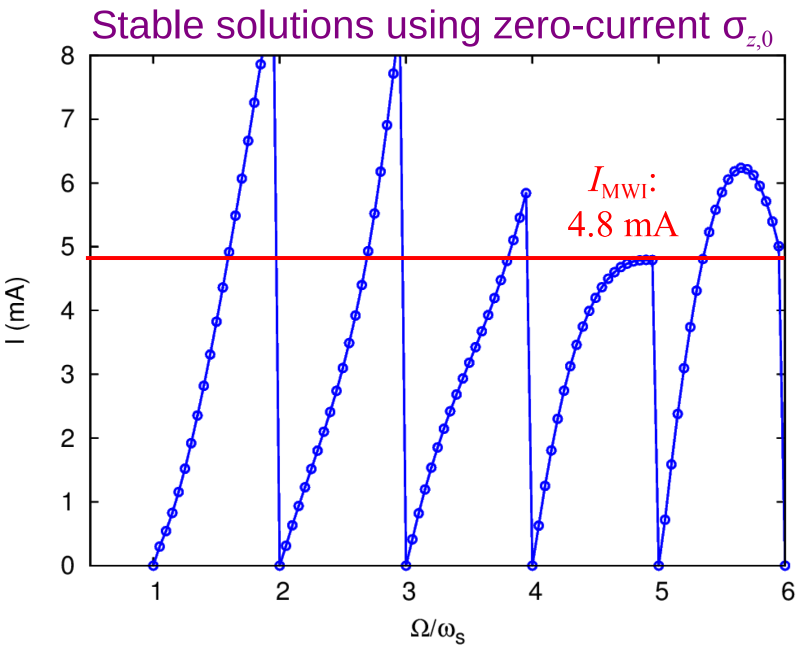

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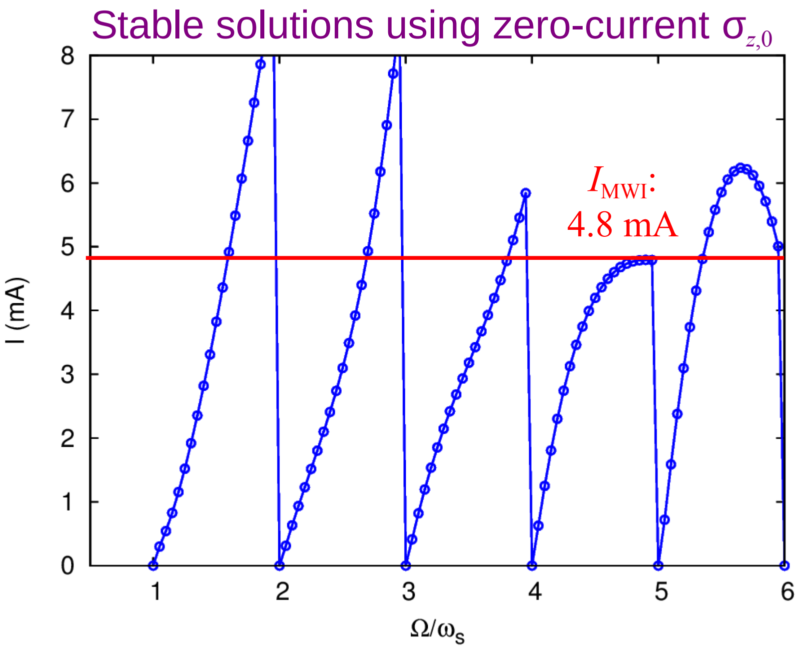

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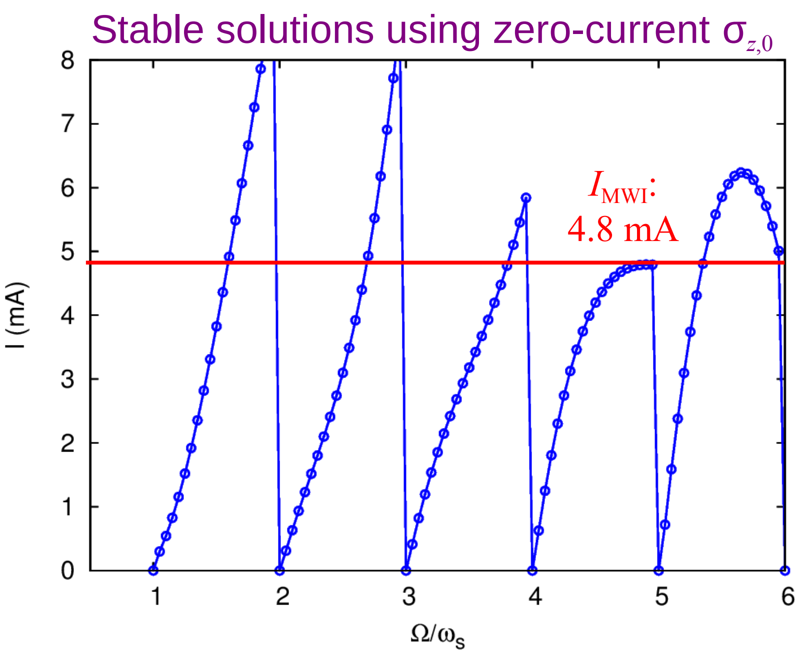

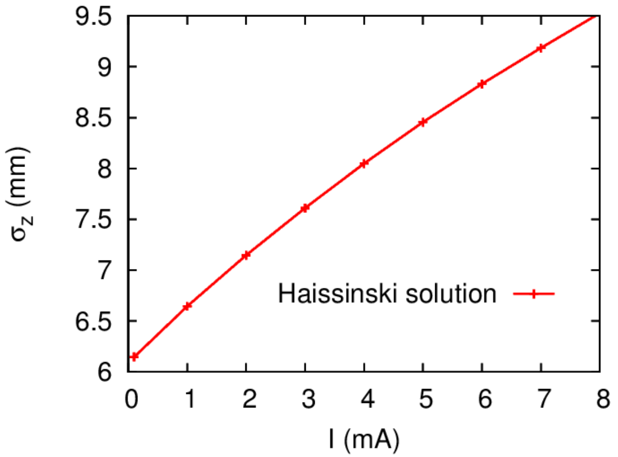

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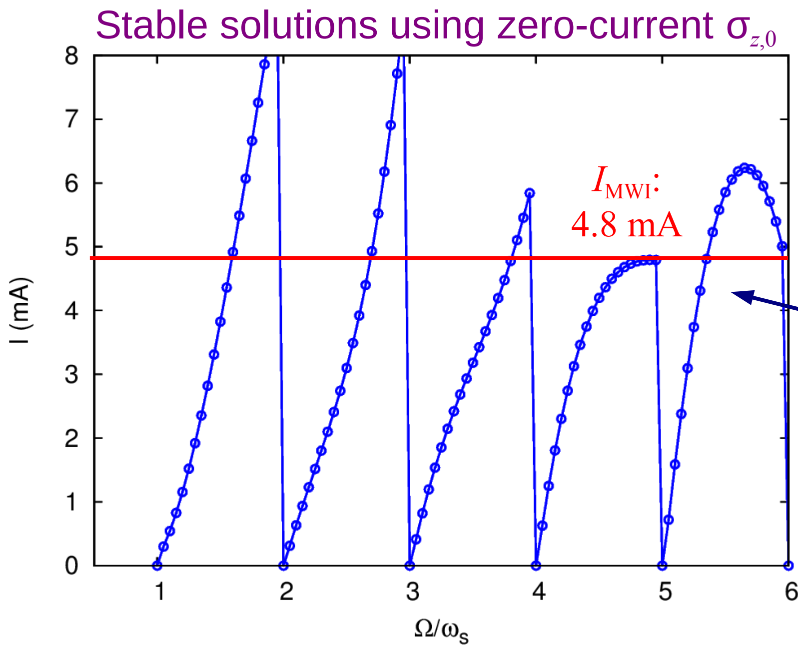

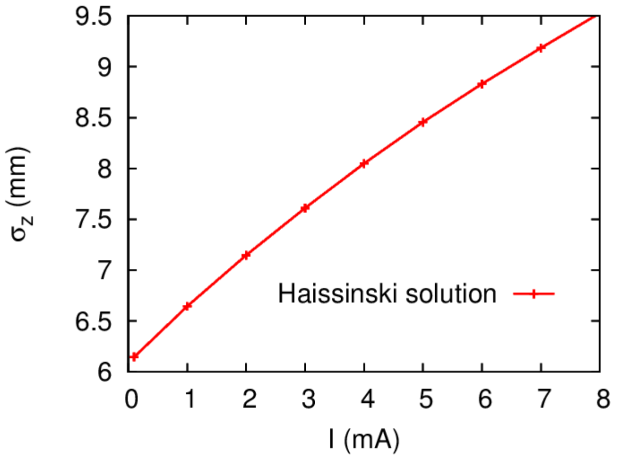

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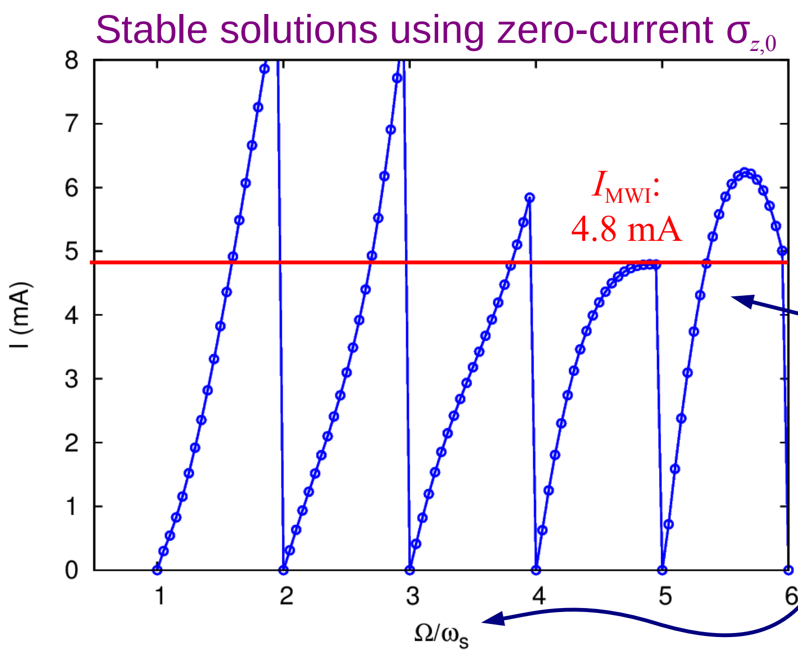

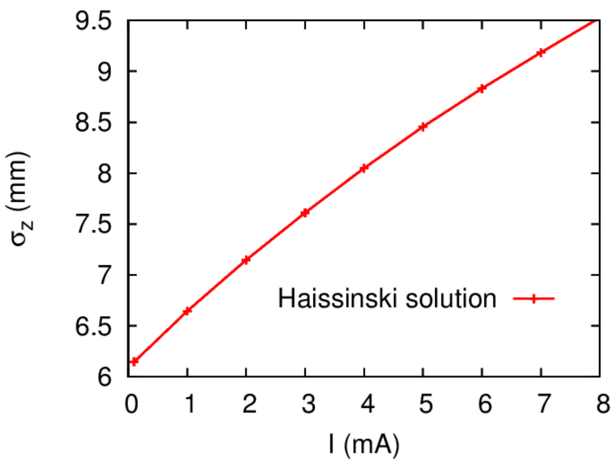

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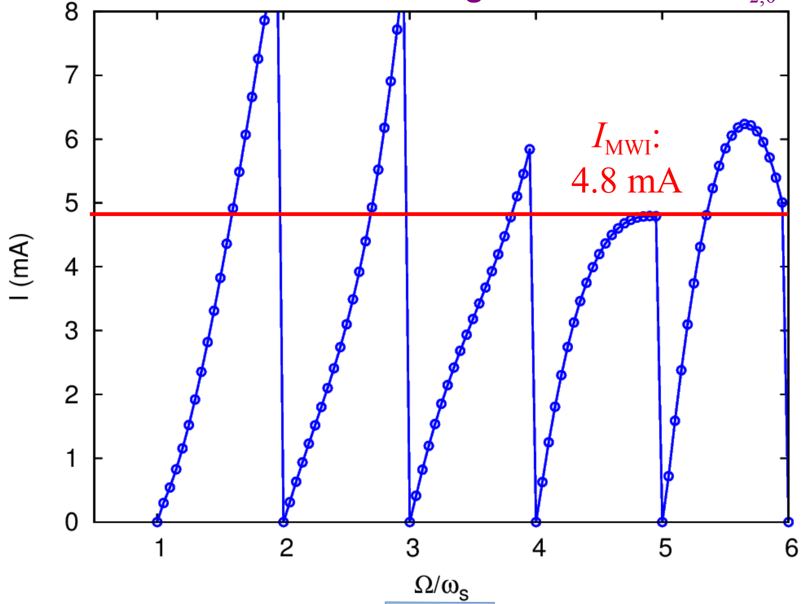
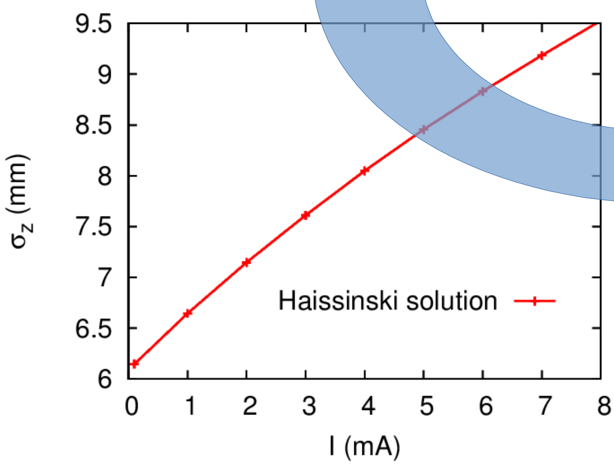
Applying the mode coupling theory to the APS

Stable solutions using zero-current $\sigma_{z,0}$





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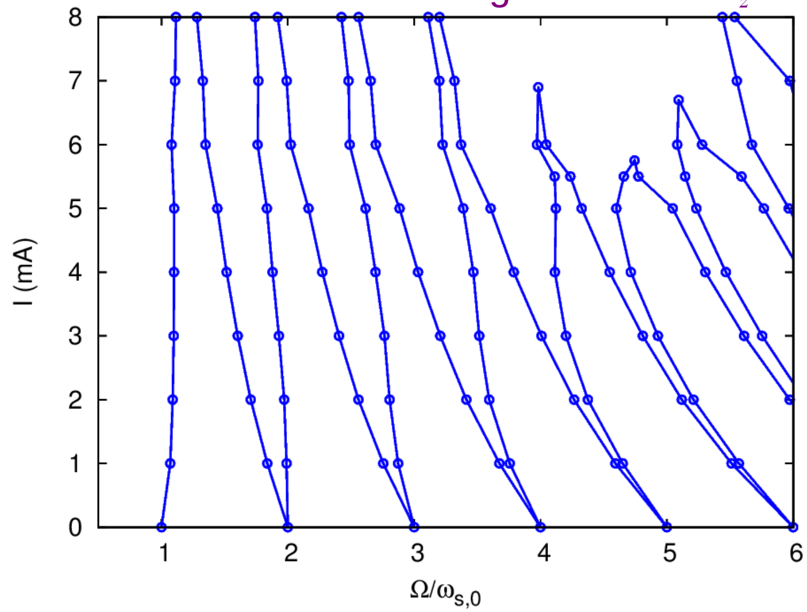

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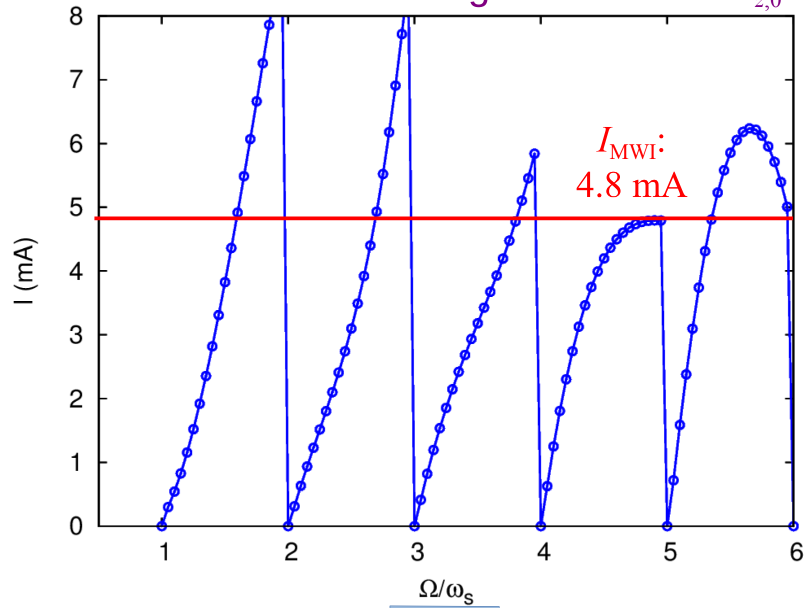

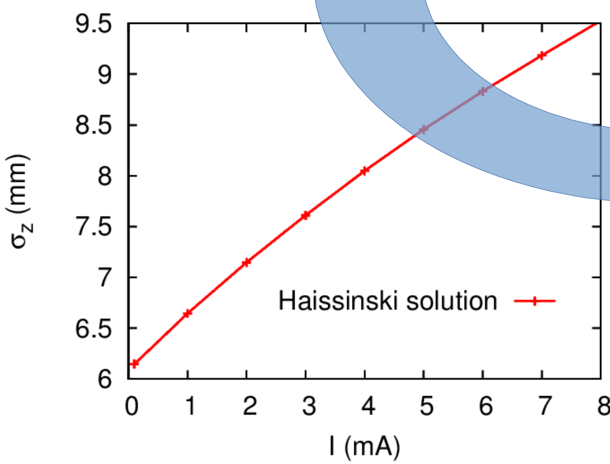

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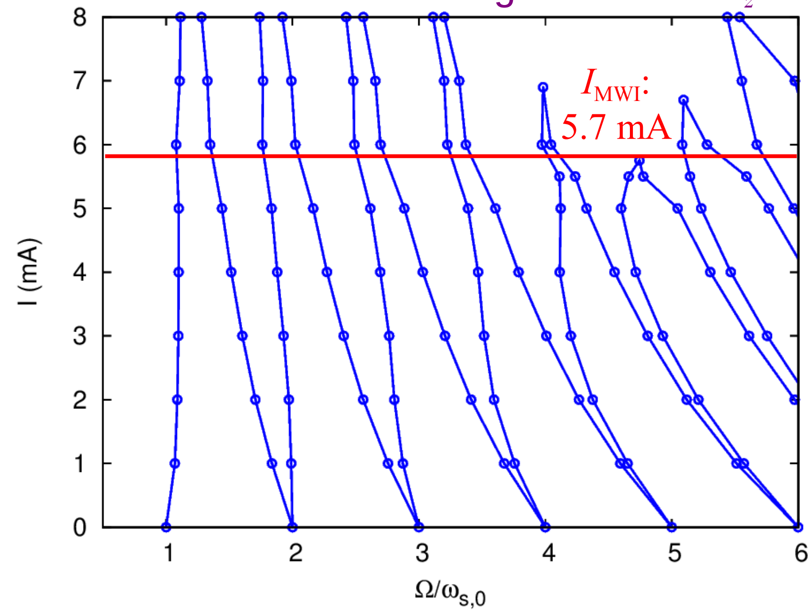
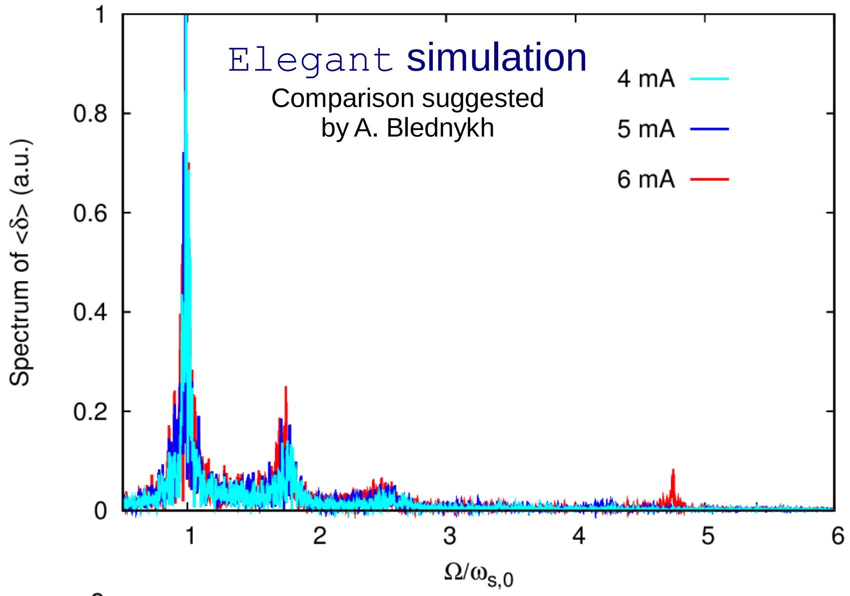
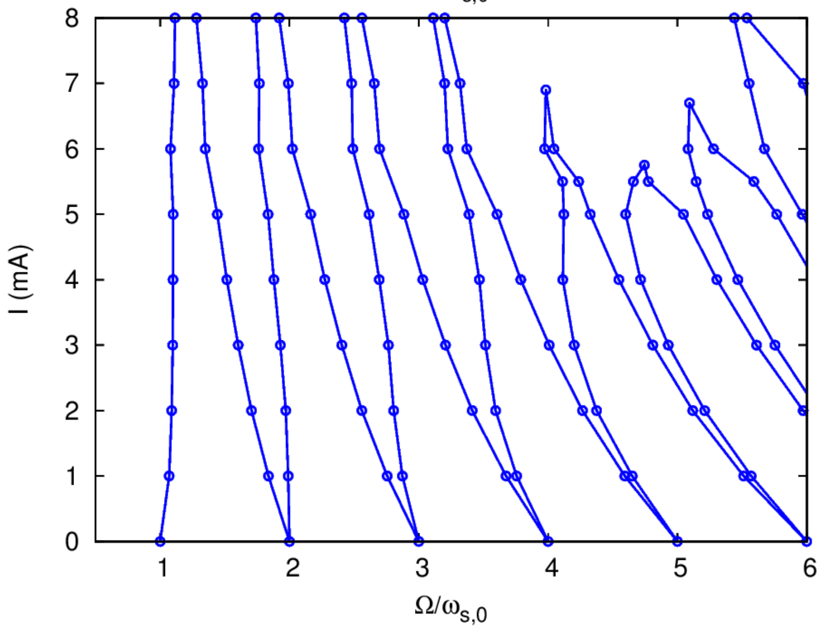


Predicted dynamics at MWI threshold





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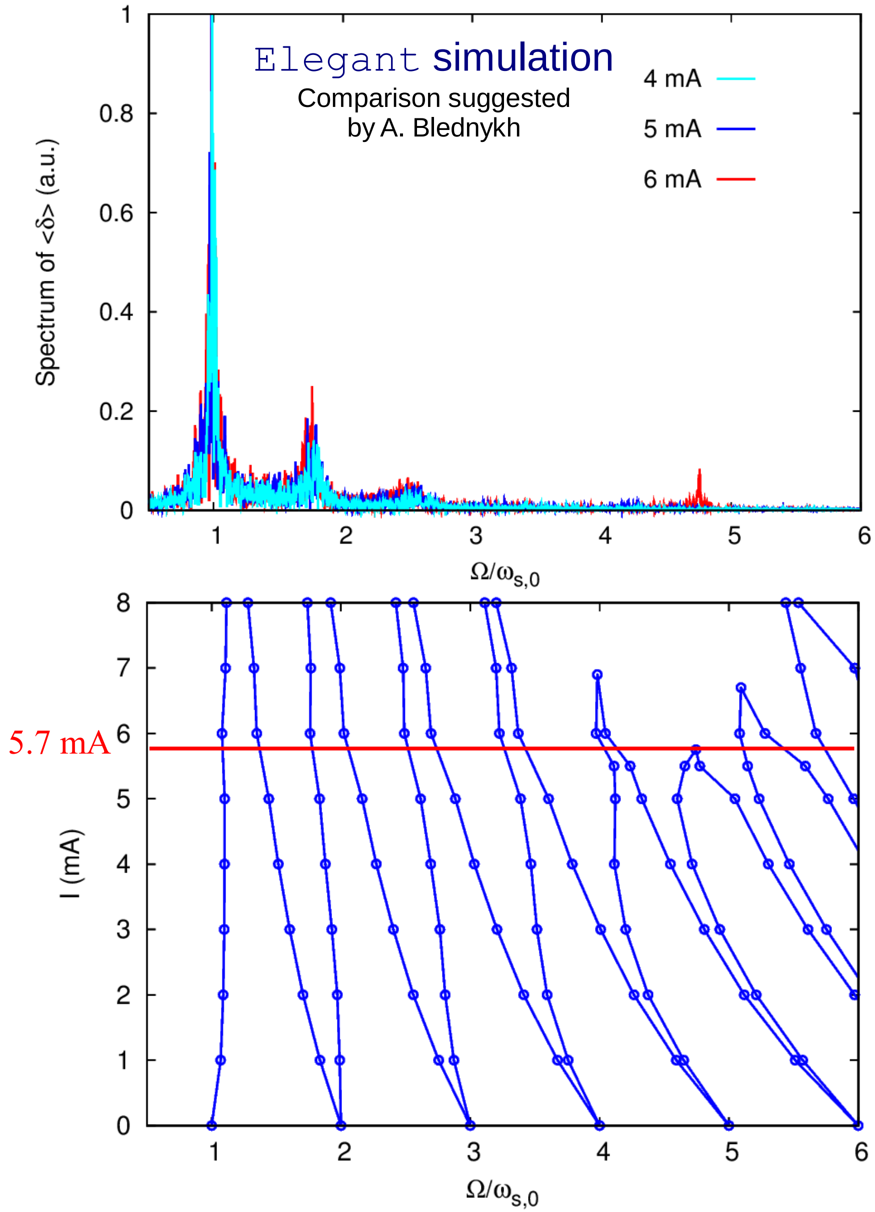

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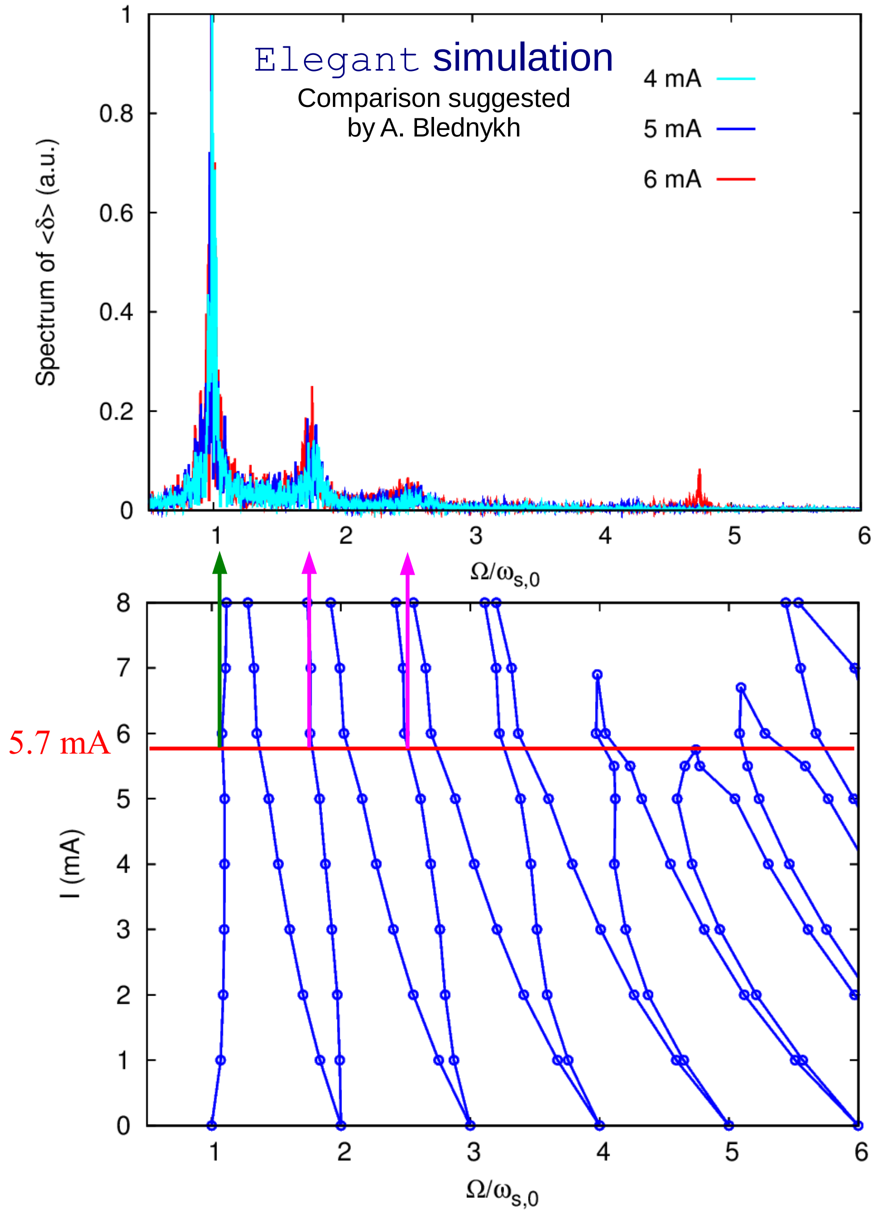
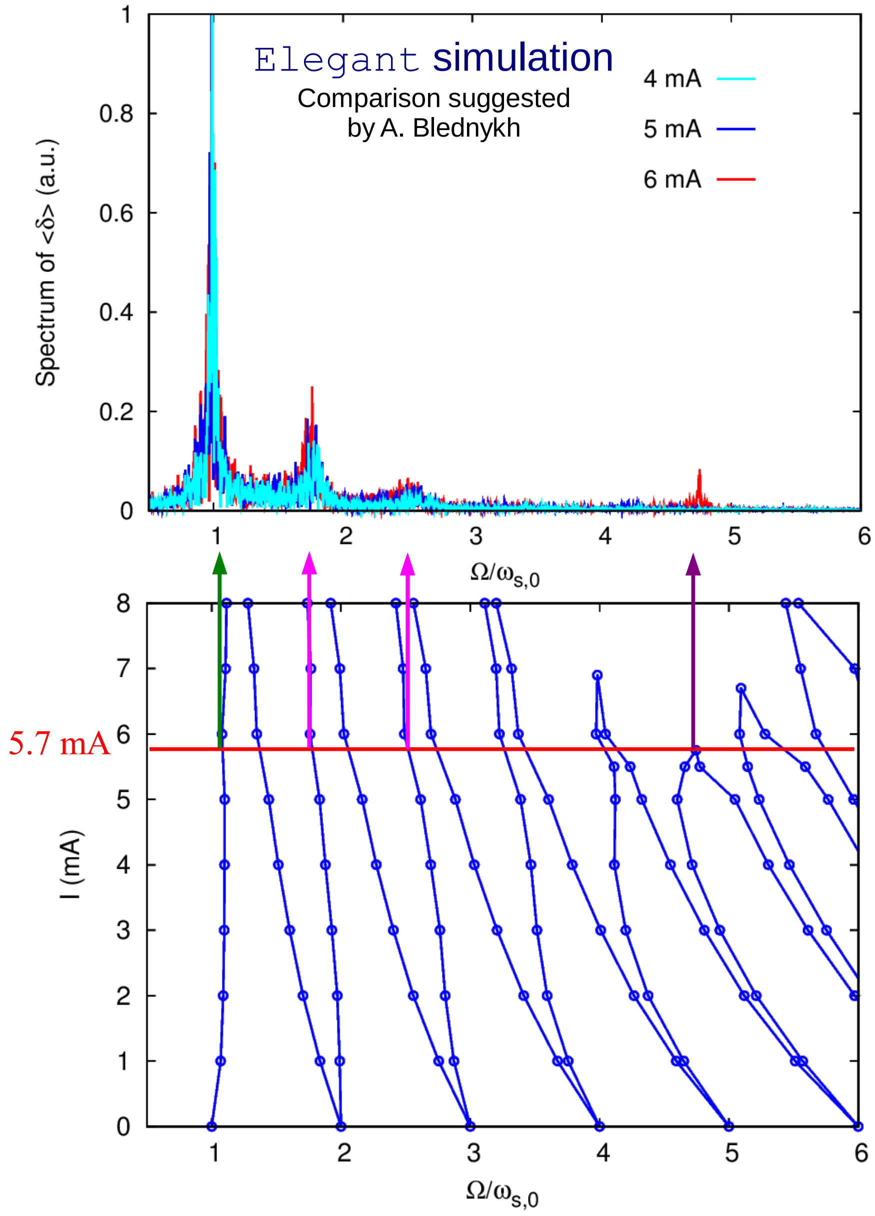


Main oscillation: $\Omega \approx \omega_{s,0}$

Harmonics: $\Omega \approx 1.7\omega_{s,0}$ and $\Omega \approx 2.4\omega_{s,0}$

Predicted dynamics at MWI threshold



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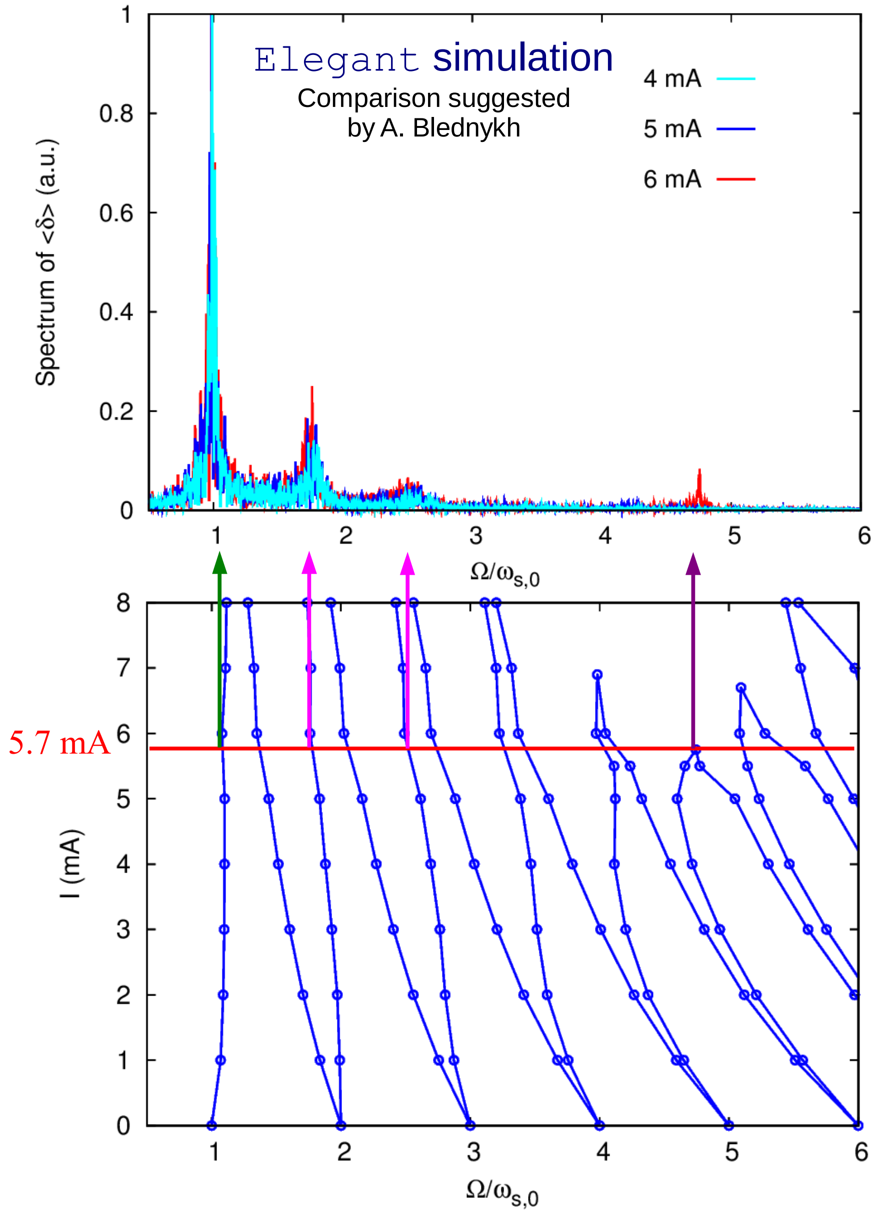

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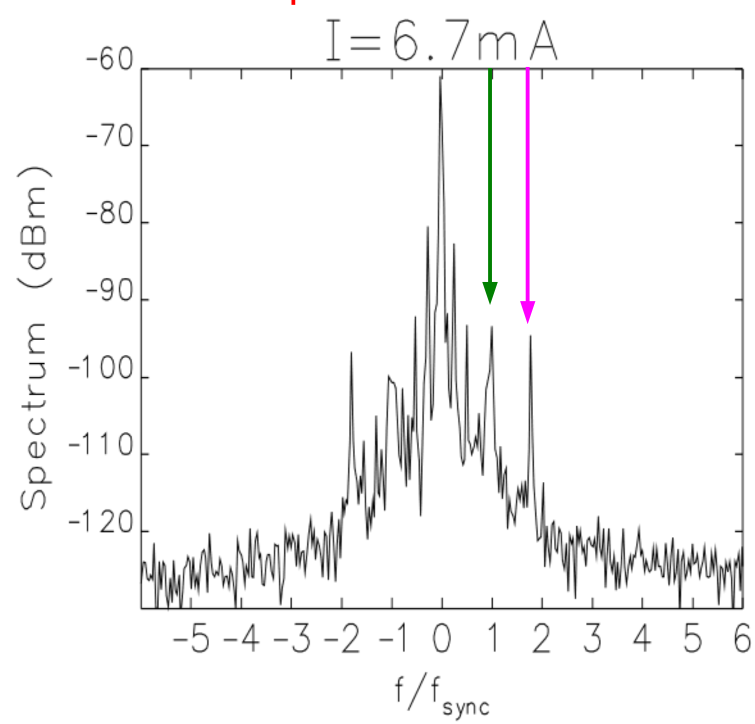

From: Y.-C. Chae, L. Emery, A.H. Lumpkin, J. Song, and B.X. Yang, "Measurement of the Longitudinal Microwave Instability in the APS Storage Ring", Proc. of PAC 2001, pp 1817.

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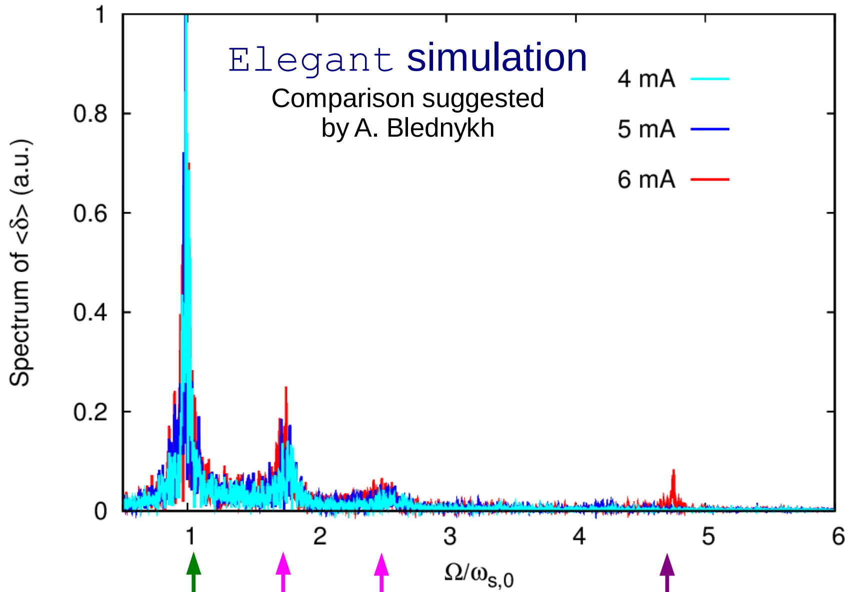

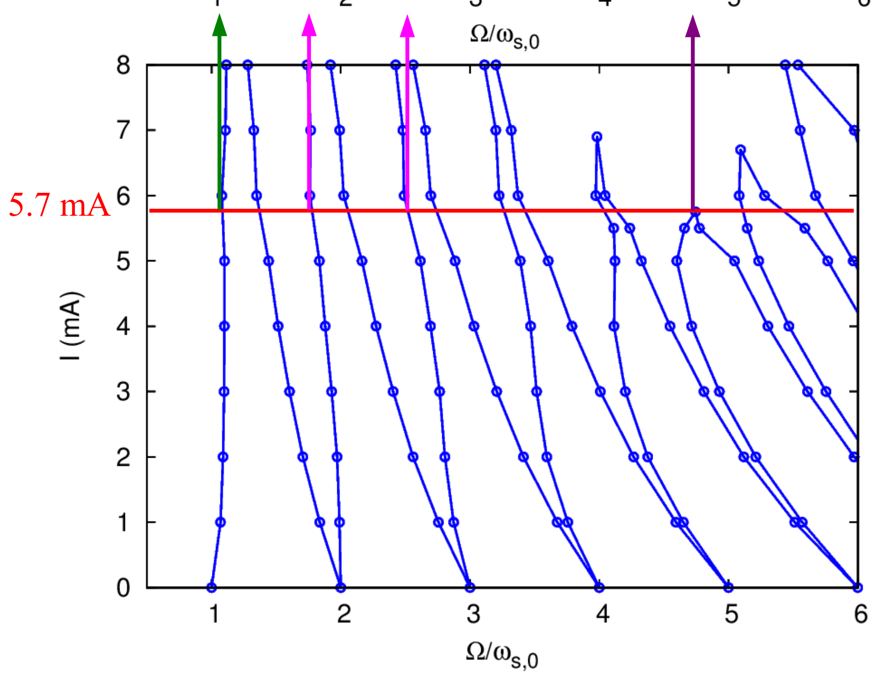

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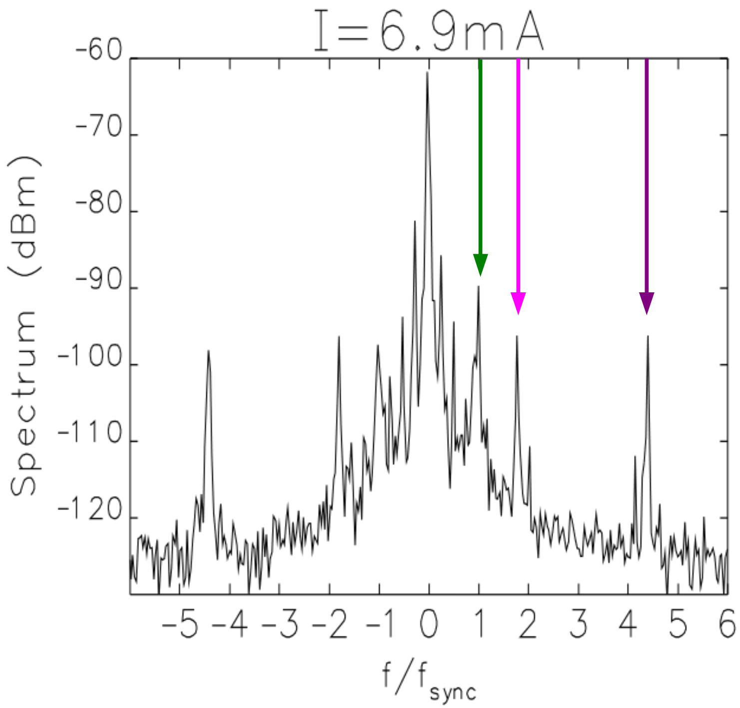

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Extension of theory to currents beyond the instability threshold

- Assume that beyond the instability threshold the energy spread increases so as to just quench the instability.
- Iterate between Haïssinski and mode-coupling theory to find self-consistent solution
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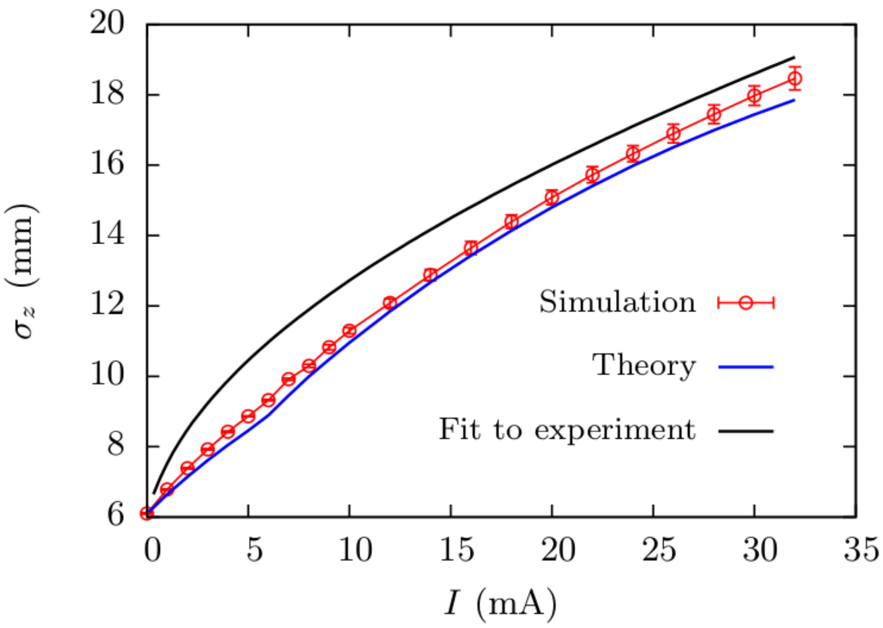

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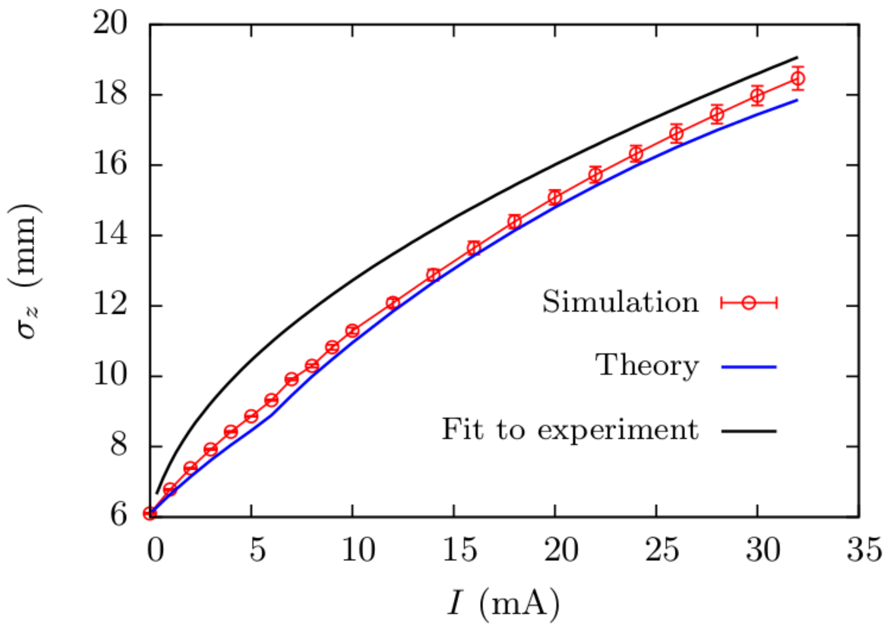

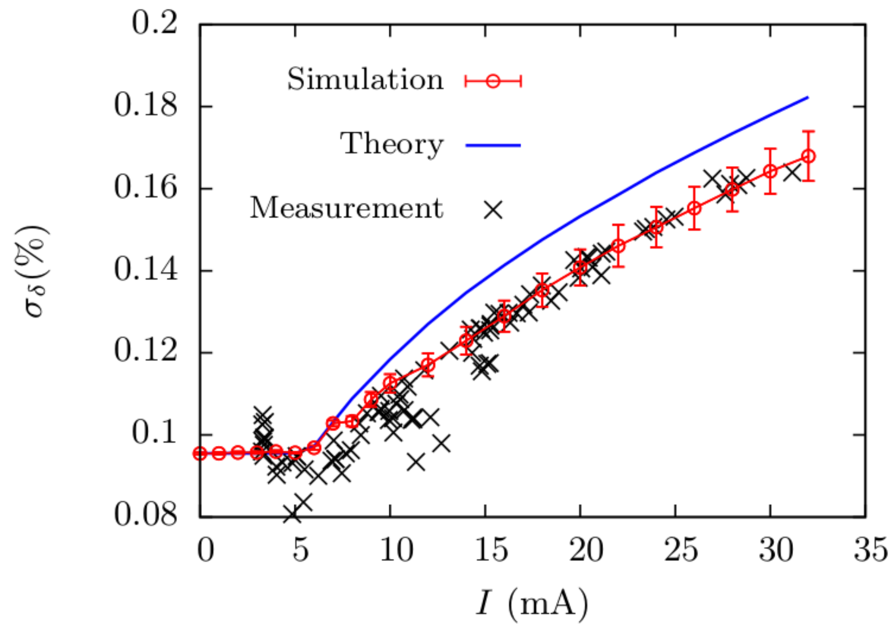

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Energy spread measurements derived from beam size in region of high dispersion and made by L. Emery and V. Sajaev at the APS in August 2014.

Conclusions & future directions

- We have developed a theoretical framework for the microwave instability that uses the mode-coupling interpretation to turn an integral equation into an eigenvalue problem
- The theory is fairly easy to solve numerically for an arbitrary impedance
- The microwave instability threshold is predicted to better than 15% for the steady-state CSR impedance, and over a wide range of broad-band resonator parameters
- The theory can be usefully applied at high intensity if one uses the Haïssinski equilibrium bunch length and an energy spread that is inflated to suppress instability
- We have found good agreement between theory, simulation, and measurements for current-dependent bunch lengthening and energy spread increase at the APS
- Extending the theory to proton machines should be easy
- Extending the theory to higher-harmonic rf systems can be done
  - Calculations will no longer be as “practical”: each matrix element will involve a numerical integral
  - Nevertheless, the theory may provide some additional insights:
    - Mode merging phenomenon will be obscured by nonlinear potential
    - We expect that the real frequencies will map out line where the growth rate equals Landau damping rate
    - We expect synchrotron radiation damping to play a role as well